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STRUCTURE OF UNIDIMENSIONAL TEMPERATURE STRESSES

A. G. Shashkov and S. Yu. Yanovskii UDC 539.32

Using the structural approach, the temperature stresses are examined in a semiinfinite rod, insulated on the lateral faces and rigidly fixed at the end. A comparative analysis is made for three heat-transfer models.

Shashkov and Abramenko [1] proposed a new, original method for describing physical phenomena. The problems of unidimensional heat conduction were examined for three heat-transfer models using the structural approach. The structural description does not require the solution of the initial determining equations. Within the scope of Laplace representations, it makes it possible to very graphically trace the formation of the physical fields and to isolate the observed coordinates.

The present work uses the structural method to describe the temperature stresses in a semiinfinite elastic rod insulated on the lateral surfaces and rigidly fixed at the end. Stresses develop in the rod as a result of the heat flux going into the fixed end. The heat transfer is described by the classic Fourier equation, by the hyperbolic equation based on the modified relationship of Vernotte–Lykov between the heat flux and the temperature gradient, and by the linearized Nunziato heat-conduction equation, which takes the thermal "memory" effects into account. A comparative structural analysis is subsequently made.

The thermoelastic state of a thin isotropic rod is described by the equation

$$\frac{\partial \sigma_{xx}(x, t)}{\partial x} = \rho \frac{\partial^2 u(x, t)}{\partial t^2} \tag{1}$$

and the relationship

$$\sigma_{xx}(x, t) = E \left(\frac{\partial u(x, t)}{\partial x} - \alpha_t T(x, t) \right). \tag{2}$$

Eliminating σ_{xx} stresses from (1) and (2), we obtain

$$\frac{\partial^2 u(x, t)}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 u(x, t)}{\partial t^2} = \alpha_t \frac{\partial T(x, t)}{\partial x}, \tag{3}$$

where $c_0 = (E/\rho)^{1/2}$ is the velocity of elastic vibrations in the rod.

The initial and the boundary conditions for the displacements have the form

$$u(x, 0) = \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = 0, \tag{4}$$

$$u(0, t) = 0; \ \lim_{x \to \infty} u(x, t) < \infty. \tag{5}$$

I. Let us show the structure of temperature stresses in a rod when the heat-transfer process is described by the classic heat-conduction equation. The temperature field is found from Fourier equation

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$$\frac{\partial \vartheta(x, t)}{\partial t} = a\frac{\partial^2\vartheta(x, t)}{\partial x^2} \tag{6}$$

under the following initial and boundary conditions:

$$\vartheta(x, 0) = 0; \ \lim_{x\to\infty}\vartheta(x, t) < \infty, \tag{7}$$

$$-\lambda \frac{\partial \vartheta(x, t)}{\partial x}\bigg|_{x=0} = q(t)\,\eta(t), \tag{8}$$

where

$$\vartheta(x, t) = T(x, t) - \theta; \ \theta = \text{const}; \ \eta(t) = \begin{cases} 1 & t \geqslant 0; \\ 0 & t < 0. \end{cases}$$

Let us seek the solution with the aid of Laplace transform. The following designation for the transformed value is used:

$$L\{f(t)\} = \int_0^\infty f(t)\exp(-pt)\,dt = \bar{f}(p). \tag{9}$$

The solution for the representation of temperature and displacement is, respectively,

$$\bar{\vartheta}(x, p) = \frac{\bar{q}(0, p)}{b\sqrt{p}}\exp(-x\sqrt{p/a}), \tag{10}$$

$$\bar{u}(x, p) = \frac{\alpha_t \bar{q}(0, p)\,[\exp(-x\sqrt{p/a}) - \exp(-px/c_0)]}{\lambda p\,(p/c_0^2 - 1/a)}. \tag{11}$$

The size of the heat flux $\bar{q}(x, p)$ in cross section x is

$$\bar{q}(x, p) = -\lambda\frac{\partial\bar{\vartheta}(x, p)}{\partial x} = \bar{q}(0, p)\exp(-x\sqrt{p/a}). \tag{12}$$

The size of deformation $\bar{\varepsilon}(x, p)$ in cross section x is determined from

$$\bar{\varepsilon}(x, p) = \partial\bar{u}(x, p)/\partial x = \left\{\alpha_t\bar{q}(0, p)\left[\frac{p}{c_0}\exp\left(-\frac{p}{c_0}x\right) - \right.\right.$$

$$\left.\left. - \sqrt{\frac{p}{a}}\exp\left(-x\sqrt{\frac{p}{a}}\right)\right]\right\} \Big/ \{\lambda p\,(p/c_0^2 - 1/a)\}. \tag{13}$$

The stresses developing in cross section x of the rod can be written in the form

$$\bar{\sigma}(x, p) = E\,[\bar{\varepsilon}(x, p) - \alpha_t\bar{\vartheta}(x, p)]. \tag{14}$$

At the fixed end of the rod the following stresses will develop:

$$\bar{\sigma}(0, p) = -\alpha_t E\bar{\vartheta}(0, p). \tag{15}$$

Using Eqs. (10)-(15), a structural chart disclosing the structure of the temperature stresses (Fig. 1a) can be constructed. It consists of two chains and has one inlet and five outlets.

The structural chart permits the solution of the following series of direct and inverse problems important in practical applications:

1. given any dependence of heat flux on time, to find the temperature field, the field of displacements, deformations, and stresses;

2. to find the principle of the heat flux change q(t) which would ensure the desired variation of stresses in the rod, in particular of stresses in the fixed end of the rod;

3. keeping track of the temperature changes, to determine the displacements, or from the experimentally determined displacements to determine the resultant stresses;

4. to determine the thermophysical characteristics and elastic constants of the material from experimentally determined fields of temperature, displacements, and stresses.

For obtaining the expression $\bar{q}(p)$, the method of heat supply to the surface of the end of the rod should be indicated, and subsequently the analytical dependence of the heat flux on the surface temperature of the end of the rod should be formulated, taking the mechanism of heat transfer to the surface into account.

II. Let us show the structure of the temperature stresses in a rod when the heat-conduction process in the rod is described by the hyperbolic heat-conduction equation [2]

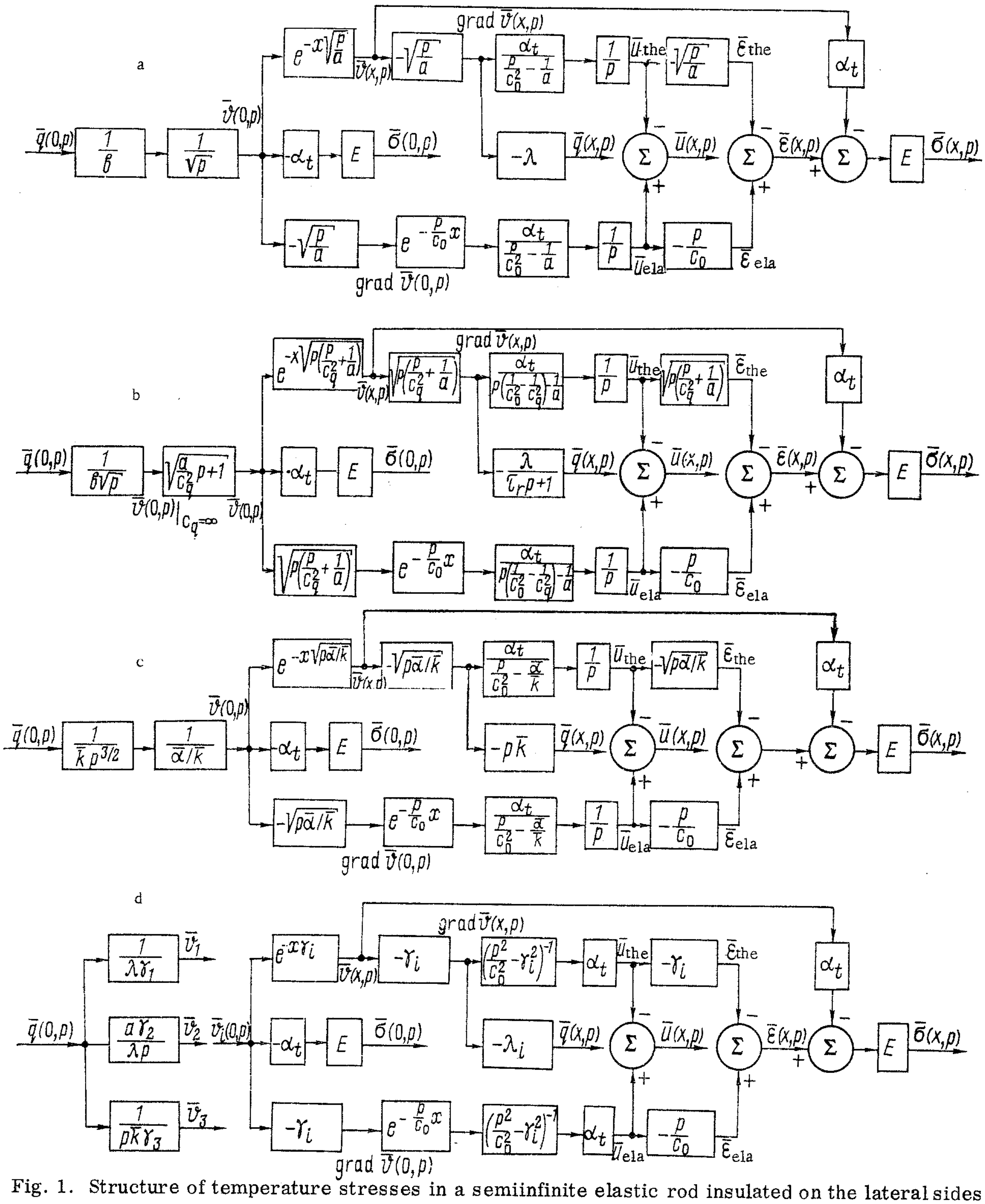

Fig. 1. Structure of temperature stresses in a semiinfinite elastic rod insulated on the lateral sides and rigidly fixed at the end: a) heat transfer described by the classic heat-conduction equation; b) by a hyperbolic equation; c) by a linearized heat-conduction equation taking the thermal "memory" of the material into account; d) general structure ($\gamma_1 = \sqrt{p/a}$; $\gamma_2 = \sqrt{p(p/c_q^2 + 1/a)}$; $\gamma_3 = \sqrt{p\bar{\alpha}/\bar{k}}$; $\lambda_1 = \lambda$; $\lambda_2 = \lambda/\tau_r p + 1$; $\lambda_3 = p\bar{k}$). In Fig. 1b $\sqrt{p[(p/c_q^2) + (1/a)]}$ members should read $-\sqrt{p[(p/c_q^2) + (1/a)]}$, and in Fig. 1c, $1/(\bar{\alpha}/\bar{k})$ should read $\sqrt{1/(\bar{\alpha}/\bar{k})}$.

$$\frac{1}{c_q^2}\frac{\partial^2\vartheta(x,t)}{\partial t^2}+\frac{1}{a}\frac{\partial\vartheta(x,t)}{\partial t}=\frac{\partial^2\vartheta(x,t)}{\partial x^2}. \quad (16)$$

The temperature and the heat-up rate at the initial moment are equal to zero:

$$\vartheta(x,0)=\left.\frac{\partial\vartheta(x,t)}{\partial t}\right|_{t=0}=0. \quad (17)$$

At infinity the temperature is limited

$$\lim_{x\to\infty}\vartheta(x,t)<\infty, \quad (18)$$

but at the end of the rod the heat flux is given as

$$-\lambda\left.\frac{\partial\vartheta(x,t)}{\partial x}\right|_{x=0}-\frac{a}{c_q^2}\left.\frac{\partial q(x,t)}{\partial t}\right|_{x=0}=q(0,t)\,\eta(t). \quad (19)$$

Applying Laplace transform to Eq. (16) and the boundary conditions (18) and (19), taking the initial condition (17) into account, we obtain the solution for temperature representation

$$\bar\vartheta(x,p)=\bar q(0,p)\,\frac{\sqrt{\frac{a}{c_q^2}p+1}}{b\sqrt{p}}\exp\left[-x\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\right]. \quad (20)$$

A solution for displacements follows from (3)-(5)

$$\bar u(x,p)=\left\{\alpha_t\bar q(0,p)\left(\frac{a}{c_q^2}p+1\right)\left[\exp\left[-x\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\right]-\right.\right.$$
$$\left.\left.-\exp\left(-\frac{p}{c_0}x\right)\right]\right\}\Big/\left\{\lambda p\left[p\left(\frac{1}{c_0^2}-\frac{1}{c_q^2}\right)-\frac{1}{a}\right]\right\}. \quad (21)$$

The heat flux and deformation $\bar\varepsilon(x, p)$ values in the cross section x of the rod will be determined as follows:

$$\bar q(x,p)=\bar q(0,p)\exp\left[-x\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\right], \quad (22)$$

$$\bar\varepsilon(x,p)=\frac{\partial\bar u(x,p)}{\partial x}=\left\{\alpha_t\bar q(0,p)\left(\frac{a}{c_q^2}p+1\right)\left[\frac{p}{c_0}\exp\left(-\frac{p}{c_0}x\right)-\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\times\right.\right.$$
$$\left.\left.\times\exp\left[-x\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\right]\right]\right\}\Big/\left\{\lambda p\left[p\left(\frac{1}{c_0^2}-\frac{1}{c_q^2}\right)-\frac{1}{a}\right]\right\}. \quad (23)$$

The stresses in cross section x and in the end of the rod are determined from equations (14) and (15).

Using expressions (20)-(23), (14), and (15), a structural chart can be constructed (Fig. 1b). Comparison of the structural charts in Fig. 1a and Fig. 1b shows that when the finiteness of the velocity of the heat propagation is taken into account, new members appear, which are described by operators $[(a/c_q^2)p+1]^{1/2}$ and $(1/p)\,[p(1/c_0^2)-(1/c_q^2)-(1/a)]^{-1}$ The first operator is also a part of the exponent index of the member in the chain, which characterizes the heat propagation in the rod, and it imparts oscillating character to the heat-conduction process. As it is seen from the structural chart, two elastic waves are already being propagated in the rod, one with the speed of sound $c_0=(E/\rho)^{1/2}$, and the second with the speed of thermal disturbances $c_q=(a/\tau_r)^{1/2}$.

Generally, all fields characterizing the thermoelastic condition of the rod can be represented in the form of two components

$$\bar f(x,p)=\bar f_{\text{the}}(x,p)+\bar f_{\text{ela}}(x,p), \quad (24)$$

where f(x, p) is the displacement, deformation, or stress.

In the classic case, as it is seen from Fig. 1a, the inlet value, which is the heat flux in the fixed end, and $\bar f_{\text{the}}$ vary synchronously, i.e., virtually at the moment of the heat-flux introduction, stress is established at once in the entire rod. The elastic part $\bar f_{\text{ela}}$ is connected with the inlet value through an operator ratio, of which the $\exp[-px/c_0]$ member is a part. Therefore, it represents an elastic wave which propagates in the rod with the speed of sound. For the hyperbolic heat-conduction equation, from the structural chart of Fig. 1b it is seen that for displacements, for example, the thermal part and the inlet value are connected through an operator ratio

$$\bar{u}_{\text{the}}(x, p) = \frac{\bar{q}(0, p)\alpha_t \left(p\frac{a}{c_q^2}+1\right)\exp\left[-x\sqrt{p\left(\frac{p}{c_q^2}+\frac{1}{a}\right)}\right]}{\lambda p\left[p\left(\frac{1}{c_0^2}-\frac{1}{c_q^2}\right)-\frac{1}{a}\right]}, \tag{25}$$

of which the member with the transfer function $\exp\left(-x\sqrt{p[(p/c_q^2)+(1/a)]}\right)$ is a part. Upon transformation it will yield an expression proportional to $\eta[t-(x/c_q)]$; i.e., the thermal part of displacements will also have an oscillating character.

As it is seen from Fig. 1b, the operator connection between the heat flux and the stress at the fixed end of the rod will have the form

$$\bar{\sigma}(0, p) = -E\alpha_t\bar{q}(0, p)\frac{\sqrt{p\frac{a}{c_q^2}+1}}{b\sqrt{p}}. \tag{26}$$

Upon completing the retransformation, we have the following function:

$$\sigma(0, t) = -E\alpha_t\int_0^t q(0, t-\tau)F(\tau)\,d\tau, \tag{27}$$

where

$$F(\tau) = \frac{c_q}{\lambda}\left[\varphi(\tau) + \tau_r\frac{\partial\varphi(\tau)}{\partial\tau}\right]; \tag{28}$$

$$\varphi(\tau) = \exp\left(-\frac{c_q^2\tau}{2a}\right)I_0\left(\frac{c_q^2\tau}{2a}\right). \tag{29}$$

If, in the course of experiment, the heat flux at the fixed end is given and the time dependence of the stress in the fixed end is obtained, then the velocity of heat propagation can be found from (27). If the velocity of heat propagation is considered to be infinitely large, then all the transfer functions in the members of Fig. 1b become identical to the members of the structure of temperature stresses in the classic case.

III. Let us show the structure of temperature stresses in a rod when the heat-conduction process is described by a differential equation taking the "thermal memory" effect into account [3]. The mathematical formulation of the problem can be expressed as

$$\alpha(0)\frac{\partial\vartheta(x, t)}{\partial t} + \int_0^\infty \alpha'(s)\frac{\partial\vartheta(x, t-s)}{\partial t}ds = k(0)\frac{\partial^2\vartheta(x, t)}{\partial x^2}\int_0^\infty k'(s)\frac{\partial^2\vartheta(x, t-s)}{\partial x^2}ds; \tag{30}$$

$$\vartheta(x, 0) = \left.\frac{\partial\vartheta(x, t)}{\partial t}\right|_{t=0} = 0; \quad \lim_{x\to\infty}\vartheta(x, t) < \infty; \tag{31}$$

$$-k(0)\left.\frac{\partial\vartheta(x, t)}{\partial x}\right|_{x=0} - \int_0^\infty k'(s)\left.\frac{\partial\vartheta(x, t-s)}{\partial x}\right|_{x=0} ds = q(0, t)\eta(t). \tag{32}$$

The equations for the field of displacements and the boundary conditions are written as (3), (4), and (5).

The solution for the temperature and displacement representations, which satisfies the boundary conditions, has a form

$$\bar{\vartheta}(x, p) = \frac{\bar{q}(0, p)}{p^{3/2}\sqrt{\bar{\alpha}/\bar{k}}}\exp\left(-x\sqrt{p\bar{\alpha}(p)/\bar{k}(p)}\right), \tag{33}$$

$$\bar{q}(x, p) = \bar{q}(0, p)\exp\left(-x\sqrt{p\bar{\alpha}(p)/\bar{k}(p)}\right), \tag{34}$$

$$\bar{u}(x, p) = \frac{\alpha_t\bar{q}(0, p)\left[\exp\left(-x\sqrt{p\bar{\alpha}(p)/\bar{k}(p)}\right) - \exp\left(-\frac{p}{c_0}x\right)\right]}{\bar{k}(p)\,p^2\left(p/c_0^2 - \bar{\alpha}(p)/\bar{k}(p)\right)}, \tag{35}$$

$$\bar{\varepsilon}(x, p) = \frac{\partial \bar{u}(x, p)}{\partial x} = \Big\{\alpha_t \bar{q}(0, p)\Big[\frac{p}{c_0}\exp\Big(-\frac{p}{c_0}x\Big) - \sqrt{p\bar{\alpha}(p)/\bar{k}(p)} \times$$

$$\times \exp(-x\sqrt{p\bar{\alpha}(p)/\bar{k}(p)}\Big]\Big\} : \{\bar{k}(p)p^2(p/c_0^2 - \bar{\alpha}(p)/\bar{k}(p))\}. \quad (36)$$

The stresses in cross section x of the rod and in the fixed end are determined from Eqs. (14) and (15).

Using expressions (33)-(36), (14), and (15), it is possible to construct a structural chart (Fig. 1c). It is seen from the chart that the structure that takes "thermal memory" into account essentially depends on the form of the nuclei of the integration operators. If relaxations of the heat flux and the internal energy do not take place, i.e.,

$$k'(t) = 0;\ k(t) = \text{const} = \lambda;$$

$$\alpha'(t) = 0;\ \alpha(t) = \text{const} = \rho c,$$

then the structure is analogous to the classic case. The same result can be arrived at if the relaxation functions are selected to be monotonically decreasing exponential functions of time, and if the solution is considered to apply to large values of time. Also, in such a case $k(t) \approx \text{const}$, and $\alpha(t) \approx \text{const}$.

Comparing the structural charts b and c of Fig. 1, one can be satisfied that when the relaxation function has a form

$$\bar{\alpha}(p) = \rho c/p;\quad \alpha(t) = \rho c\eta(t);$$

$$\bar{k}(p) = \lambda/p(\tau_r p + 1);\quad k(t) = \lambda\Big(1 - \exp\Big[-\frac{t}{\tau_r}\Big]\Big),$$

then the structural chart of temperature stresses taking "thermal memory" effects into account takes on the form of the structural chart that describes the temperature stresses for the hyperbolic heat-conduction equation. Generally speaking, when the "thermal memory" effects are taken into account, a second wave may arise in the rod depending on the form of the relaxation nuclei.

As it is seen from Fig. 1c, if the dependences of stresses and fluxes on time in the fixed end of the rod, and the temperature gradients and the heat fluxes in the cross section x of the rod are determined experimentally, then the relaxation functions of the heat flux and the internal energy can be calculated:

$$k(t) = L^{-1}\Big\{-\frac{1}{p}\,\frac{\bar{q}(x, p)}{\operatorname{grad}\bar{\vartheta}(x, p)}\Big\}, \quad (37)$$

$$\alpha(t) = L^{-1}\Big\{-\frac{\alpha_t^2 E^2}{p^2}\,\frac{\bar{q}^2(0, p)}{\bar{\sigma}^2(0, p)}\,\frac{\operatorname{grad}\bar{\vartheta}(x, p)}{\bar{q}(x, p)}\Big\}. \quad (38)$$

In generalizing the obtained data, let us construct a structural chart of temperature stresses in an elastic rod (Fig. 1d). As it is seen, the chain that shows the formation of the temperature field and the thermal part of the temperature stresses, deformations, and displacements includes the value $\gamma_i(p)$, which characterizes the heat-transfer model, in the transfer functions of the members. The chain that characterizes the origination of the elastic displacement component also includes the value $\gamma_i(p)$ in the transfer functions of the members, since the equation for displacements is associated with the temperature gradient. If rods made of viscoelastic materials were examined, then this chain would include parameters which, when changed, would yield various models of viscoelastic behavior.

The structure of temperature stresses permits a graphic tracking of the formation of the stressed state in the rod. The heat flux upon entering the fixed end heats it up and forms a temperature gradient grad $\bar{\vartheta}(0, p)$. Compression of the subsurface layer and a further propagation of the elastic wave take place with a velocity c_0. This is indicated by the member with the transfer function $\exp[-p(x/c_0)]$ in the chain, which shows the formation of the elastic displacement components. The thermal-displacement components $\bar{u}_{the}$, as it is seen from the structure, are proportional to the temperature gradient in the cross section x of the rod. The formation of grad $\bar{\vartheta}(x, p)$ depends on parameter $\gamma_i(p)$, i.e., on the heat-transfer model. In the classic case $\gamma_i = \sqrt{p/a}$ and the temperature gradient grad $\bar{\vartheta}(x, p)$ is established simultaneously with the supply of heat to the fixed end, i.e., the thermal components have a diffusional character. For the other two cases, the formation of grad $\bar{\vartheta}(x, p)$ is retarded, i.e., the thermal components are formed in the form of a disturbance which is being propagated with a certain velocity characterizing the propagation of thermal disturbances.

NOTATION

k(t), heat flux relaxation function; $\alpha(t)$, internal energy relaxation function; T, rod temperature; θ, ambient temperature; t, time; x, coordinate along the rod; $\sigma_{xx}(x, t)$, stress; u(x, t), displacement; $\varepsilon(x, t)$, deformation; $c_0 = (E/\rho)^{1/2}$, speed of sound in the rod under isothermal conditions; E, elasticity modulus; ρ, density of the material; α_t, coefficient of thermal expansion; λ, thermal-conductivity coefficient; a, thermal-diffusivity coefficient; b, thermal-activity coefficient; $c_q = (a/\tau_r)^{1/2}$, velocity of heat propagation; τ_r, heat flux relaxation time; $\eta(t)$, unique Heaviside function.

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HYDRODYNAMICS AND HEAT AND MASS TRANSFER IN A SOLIDIFYING MELT

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The effect of natural thermal convection in a solidifying melt on the distribution of an admixture in the liquid and solid phases as a function of the Lewis and Schmidt numbers is studied numerically.

It is known [1-3] that the hydrodynamics of the melt has an important influence on the processes of heat and mass transfer in a crystallizing ingot. Therefore, in an analysis of the conditions of formation of chemical nonuniformity of ingots it is important to know the laws of the development of the convective currents and the distribution of the velocity fields in the volume of unsolidified metal.

There are several natural causes producing the mixing of the liquid metal during its crystallization:

1) natural thermal convection of the melt owing to its temperature nonuniformity;

2) shrinkage of the metal during solidification caused by the difference in densities of the liquid and solid phases;

3) concentration convection due to the nonuniformity of the concentration of the admixture in the melt;

4) the formation of nuclei of the solid phase at the crystallization front and their descent along the crystallization boundary.

One of the main causes of mixing of the melt in the process of its crystallization is natural thermal convection [1], the investigation of which is a complex and important problem.

The problem of the distribution of an admixture in a solidifying melt is solved for the most part in a one-dimensional formulation and without allowance for thermal convection, which is evidently connected with the absence of analytical methods for solving nonsteady nonlinear differential equations of transfer of an admixture in regions with moving boundaries under conditions of convective motion of the solidifying melt. In this connection the influence of natural thermal convection on the distribution of an admixture in a solidifying melt has been studied insufficiently and requires additional research.

A rectangular region, semiinfinite along the horizontal coordinate normal to the plane of the cross section, is analyzed for a study of thermal convection and its influence on the processes of transfer of an admixture in a solidifying melt. The region is filled with a melt (of low-carbon steel) with an initial temperature

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